4/17/18 "Dont cry because its over, smile because it happened."-Dr. Seuss

HW: "Sequences" homework section

AIM: What are Sequences?

Warm Up:

Pattern

Sequences are extremely important in mathematics, both theoretical and applied. A **sequence** is formally defined as **a function that has as its domain the set the set of positive integers**, i.e. $\{1, 2, 3, ..., n\}$.

***Exercise* #1:** A sequence is defined by the equation $a(n) = 2n - 1$.

(a) Find the first three terms of this sequence, denoted by a_1, a_2, and a_3.

$a(1) = 2(1) - 1 = 1$

$a(2) = 2(2) - 1 = 3$

$a(3) = 2(3) - 1 = 5$

$\boxed{1, 3, 5}$

(b) Which term has a value of 53?

$53 = 2n - 1$

$+1 \qquad +1$

$\frac{54}{2} = \frac{2n}{2}$

$27 = n$

(c) Explain why there will not be a term that has a value of 70.

$70 = 2n - 1$

$+1 \qquad +1$

$\frac{71}{2} = \frac{2n}{2}$

$35.5 = n$

Not an integer

Recall that sequences can also be described by using **recursive definitions**. When a sequence is defined recursively, terms are found by operations on previous terms.

recursive: you need to know the previous term(s).

***Exercise* #2:** A sequence is defined by the recursive formula: $f(n)=f(n-1)+5$ with $f(1)=-2$.

previous term

(a) Generate the first five terms of this sequence. Label each term with proper **function** notation.

First term is -2

$f(1)=-2$

$f(2)=f(2-1)+5$
$=f(1)+5$
$=(-2)+5=3$

$f(3)=f(3-1)+5$
$=f(2)+5$
$=3+5=8$

$f(4)=8+5=13$

$f(5)=13+5=18$

(b) Determine the value of $f(20)$. Hint – think about how many times you have added 5 to **−2**.

$f(20)=-2+5(20-1)$
$=-2+5(19)$
$=-2+95$
$=93$

***Exercise* #3:** Determine a recursive definition, in terms of $f(n)$, for the sequence shown below. Be sure to include a starting value.

5, 10, 20, 40, 80, 160, …

$f(1)=5$

Recursive Formula: $f(n)=2\cdot f(n-1)$

***Exercise* #4:** For the recursively defined sequence $t_n=(t_{n-1})^2+2$ and $t_1=2$, the value of t_4 is

(1) 18 (3) 456

(2) 38 (4) 1446

square the previous term and add 2

$t_1=2$

$t_2=(2)^2+2=6$

$t_3=(6)^2+2=38$

$t_4=(38)^2+2=1446$

***Exercise* #5:** One of the most well-known sequences is the Fibonacci, which is defined recursively using two previous terms. Its definition is given below.

$$f(n) = f(n-1) + f(n-2) \text{ and } f(1) = 1 \text{ and } f(2) = 1$$

Generate values for $f(3)$, $f(4)$, $f(5)$, and $f(6)$ (in other words, then next four terms of this sequence).

$f(1) = 1$
$f(2) = 1$
$f(3) = 1 + 1 = 2$
$f(4) = 2 + 1 = 3$
$f(5) = 3 + 2 = 5$
$f(6) = 5 + 3 = 8$

1 1 2 3 5 8

It is often possible to find algebraic formulas for simple sequences, and this skill should be practiced.

***Exercise* #6:** Find an algebraic formula $a(n)$, similar to that in *Exercise* #1, for each of the following sequences. Recall that the domain that you map from will be the set $\{1, 2, 3, ..., n\}$.

(a) 4, 5, 6, 7, ... $\boxed{a(n) = n + 3}$

$4 = 1 + 3$
$5 = 2 + 3$
$6 = 3 + 3$
$7 = 4 + 3$

(b) 2, 4, 8, 16, ... $\boxed{a(n) = 2^n}$

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$

(c) $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, ...$ $\boxed{a(n) = \frac{5}{n}}$

(d) −1, 1, −1, 1, ... $\boxed{a(n) = (-1)^n}$

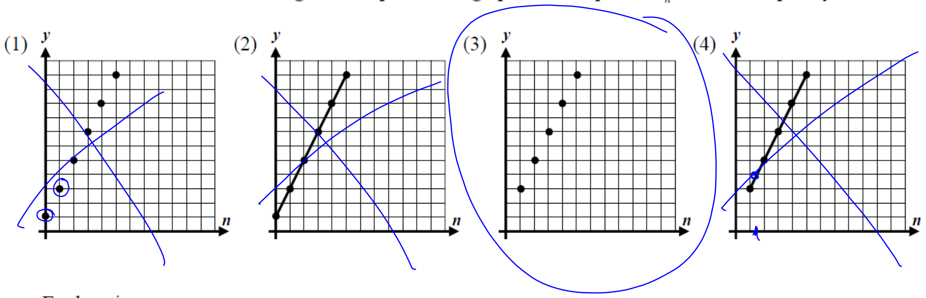
(e) 10, 15, 20, 25, ... $\boxed{a(n) = 5n + 5}$

$5 + 5(1)$
$5 + 5(2)$

(f) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, ...$ $\boxed{a(n) = \frac{1}{n^2}}$ OR $\boxed{a(n) = n^{-2}}$

$a(n) = 2n + 1$

***Exercise* #7:** Which of the following would represent the graph of the sequence $a_n = 2n + 1$? Explain your choice.



Explanation:

Domain is positive INTEGERS!